Influence of Electric Assistance Steering System on Vehicle

Body Rolling Stability

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Abstract This research is projected to analyze the steering system dynamic characteristics and vehicle body rolling stability of conventional and electric power assisted steering have been performed to deduce the relation between steering system performance and vehicle body roll stability for different operating conditions. Eight degrees of freedom model for rack and pinion steering system has been used for conventional and electric power assisted on the column steering system with PD control, and their effect on the vehicle stability of its lateral, yaw, and body roll motion specially. The model has been excited through unit torque on the steering hand wheel. A response comparison has been performed between the two steering systems. The results analysis illustrated that the electric power assisted steering with PD controller generates quieter response dynamics at high frequency response in all steering model elements than that unassisted system. The vehicle body roll torque of the assisted steering has smoother response within comfortable ride frequency range and low signal magnitude within 65% to 70% of those values of the unassisted system as a damping effect of the steering electric motor. Consequently, this assisted steering system is safer for the vehicle lateral stability response, good ride comfort, and longer service life of the steering mechanism and suspension system elements under quitter response of assisted electric motor damping effect.

1. Introduction

The steering system is one of major subsystems for vehicle operation safety and stability dynamics to negotiate a road curve or corner smoothly with low and appropriate driver effort and makes maneuvering a vehicle much easier, comfortable and safe[1]. It turns the front wheel plane to the desired direction set by the driver’s steering inputs of torque, angle rotation and speed under self-aligning moment effect.

The Electric Power Assist Steering (EPAS) is more efficient than the other power steering systems, since the electric power steering motor only supplies its assist when the steering wheel is turned. An advanced steering system has been developed to completely away of the steering column and shaft[2].

The vehicle suspended body lateral rolling includes the variations in its wheel path, suspension dimensions and parameters, instantaneous CG point and tyre stiffness according to the effect of lateral force acting on the vehicle wheels[3]. Consequently, any improvement in the steering angles dynamic response will control the lateral response to avoid vehicle lateral skid or overturn within very short response time.

The front wheels steering restores torque which tends to return the wheels to its original position arises in accordance with the wheel location angles such as king pin inclination, camber, and caster, beside the effect of tire rolling resistance direction.
Although this restoring torque provides steering stability, the driver must provide sufficient torque to overcome this required torque to steer the vehicle into the desired direction.

1.1 Coordinate Systems of Vehicle Motion

The basic concepts of the vehicle motions coordinate systems used in this work will be presented in Fig.(1) according to the ISO coordinate systems which are based on the universal coordinate systems as follows [5]:

Fig.(1): Vehicle motions coordinate systems [5]

X, Y, and Z represent the global coordinate system to describe the entire environment of the model. It is used as the center of gravity position reference for the vehicle because of the global coordinate system which does not move during simulation. Yaw is the rotation around the vertical axis through the center of gravity of the vehicle, while the Pitch motion is the rotation around the lateral axis (y) through the center of the vehicle gravity. The Roll is the rotation motion around the longitudinal x-axis through the center of gravity of the vehicle. This rotation can be felt during lateral acceleration of the vehicle.

2. Steering System Dynamics and Modulation

An electric power assisted steering system has been described, modeled and analyzed. The steering system has been modeled to include steering system geometry and steering road wheel feedback torque under effect of power assist and wheel self-alignment moment dynamics. The Steering geometry includes the transmission system from the driver hand wheel angle as an input to the road wheel angle response according to its system mechanism design and reduction ratio. Figure (2) shows five degrees of freedom model for the rack and pinion with column power assisted steering system.

The system equations of motions have been formulated according to the following assumptions [1]:

1. Linear stiffness of rack, tie rod and tyre
2. Neglected clearance between pinion and rack
3. Stiff tie-rods and neglect friction of joint elements
4. Neglected sides slip angles and damping of tire lateral deflection.
5. Neglected elasticity of pinion and reduction gear tooth
6. Constant tyre inflation pressure.
7. Correct fitting of the wheel on the sub-axle.

A physical model of column electric assisted steering system includes a steering hand wheel dynamic angle $\theta_s$, shaft stiffness $K_s$ and external damping coefficient acting on the steering shaft $C_s$. Steering driver's torque $T_d$ is transmitted from the hand wheel through the steering shaft and column into pinion at the second end of the column. A DC motor is attached to the column through a reduction unit to give assisted torque $T_m$. A control unit is used to supply the DC drive motor by the required command of volt and current. The pinion transmits the
angular motion of the steering column into a lateral motion through the rack, which has the same pitch of the pinion.

Fig. (2): Model scheme of electric assisted steering system

The main difference between this system and the conventional mechanical system is the electric motor and control parameters. As shown the model consists of steering hand wheel with radius $R_{hw}$, which is attached to the steering shaft. The column has a stiffness $K_c$ and damping coefficient $C_s$. The pinion has a radius $r_p$, it transmits the angular motion of the steering column into lateral motion of the rack, which has the same pitch of the pinion with relative neglected tooth angle effect. The rack is attached to the tie-rods through a ball joint. There is a damping for the tie-rod joints and rack movement guides with coefficient $C_r$. The tie rods are fastened to the road wheels through ball joint, and have stiffness $K_r$. The differential equation of the hand steering wheel is,

$$ J_s \ddot{\theta}_s + C_s \dot{\theta}_s + K_s (\theta_s - \theta_c) = T_d \hspace{2cm} \text{(1)} $$

Where $J_s = \frac{1}{2} m_{hw} R_{hw}^2 + \frac{1}{2} m_s r_s^2$

$m_{hw}$ and $m_s$ are masses of the hand wheel and steering shaft. $R_{hw}$ and $r_s$ are the radii of the hand wheel and its steering shaft.

The differential equation of the steering column torque can be expressed as a function of the column parameters $C_c$, $K_c$, and equivalent stiffness of the rack and tie rod $K_r$ under the torque effect of the electric motor $T_m$ as follows;

$$ J_c \ddot{\theta}_c + C_c \dot{\theta}_c + K_s (\theta_s - \theta_c) + K_c (\theta_c - \frac{2 \pi}{r_p}) = T_m \hspace{2cm} \text{(2)} $$

Where $J_c = J_e + N^2 J_m$, $J_e = \frac{1}{2} m_c r_c^2$
The PD controller gain parameters are tuned according to the values of $k_p$ and $k_d$ to ensure maximum controller performance at relative high frequency, smooth and quiet response. The mean squared errors technique of the actual motor current $i_a$ and its predicted value can be used to form the objective function of the response optimization process. Figure (3) shows a block diagram for the used analyzed electric power assisted steering system, which used for more than 6 Nm steering moment on the driver wheel.
The differential equation of the rack axial motion $x_r$ can be written as a function in its mass $m_r$, equivalent damping coefficient $C_r$, pinion to rack friction efficiency $\eta_f$, steering linkage stiffness $K_{rs}$ and its transmission efficiency $\eta_b$ for steering angles $\delta_o$ and $\delta_i$ for the outer and inner wheel angles as follows;

$$m_r \ddot{x}_r + C_r \dot{x}_r + \eta_f \frac{K_c}{r_p} \left( \frac{x_r}{r_p} - \dot{\theta}_c \right) + 2 \eta_b K_{rs} \left( \frac{x_r}{r_s} - \frac{\delta_o}{l} - \frac{\delta_i}{l} \right) = 0 \quad \text{(6)}$$

Each steerable tire has a moment of inertia $J_w$, damping coefficient $C_w$ and stiffness factor $K_w$ with knuckle steering arm $L_a$, and alignment steering moments $AT_i$ and $AT_o$ for the inner and outer wheels with steering angles $\delta_{wi}$ and $\delta_{wo}$. The differential equations of the tires motions can be expressed for the inner one as:

$$J_w \ddot{\delta}_{wi} + C_w \dot{\delta}_{wi} + K_w \delta_{wi} + K_{sl} \left( \frac{\delta_{wi}}{L_a} - \frac{2x}{L_a} \right) + AT_i = 0 \quad \text{(7)}$$

And for the outer steered wheel, the differential equation can be expressed as:

$$J_w \ddot{\delta}_{wo} + C_w \dot{\delta}_{wo} + K_w \delta_{wo} + K_{sl} \left( \frac{\delta_{wo}}{L_a} - \frac{2x}{L_a} \right) + AT_o = 0 \quad \text{(8)}$$

Where the steerable wheel moment of inertia can be determined as function in its width $d$, material density and its outer radius $R$,

$$J_w = 8 \rho \left[ \frac{1}{48} d^3 R^2 \frac{\pi}{2} + \frac{1}{24} d \frac{3 R^4 \pi}{2} \right] \quad \text{(9)}$$
The tire stiffness can be determined as function in its maximum adhesive torque, where the angle $\theta_1$ is the maximum tire twist under maximum adhesion torque.

$$Kw = F_z \phi \frac{d}{\theta_1 \sqrt{8}} \quad \text{..................................................(10)}$$

The automobile steering system should be designed carefully for suitable frequency range performance to steer the vehicle wheel at low steering resistance and driver efforts. Consequently, different mechanical steering mechanisms are used for passenger cars to convert the steering hand wheel angle to the road steerable wheel and to convey feedback about the vehicle's state of movement back to the hand steering wheel [1].

3. Vehicle Lateral Stability

The lateral dynamics of a vehicle varies in its response according to the variations of vehicle center of gravity, mass, suspension parameters, yaw moment of inertia, tire lateral stiffness and road surface adhesion, and the road surface inclination. All of the given parameters change depending on the vehicle load distribution, tires, and road conditions, which control the stability parameters instantaneous variations [7]. The vehicle body side slip angle and its roll angle can be stabilized of controlled yaw rate [8], where a single vehicle accident from rollover due to sharp turn at relative high speed, which causes large lateral acceleration of the vehicle CG. Consequently, the rollover of the vehicle can be avoided through lowering its velocity and yaw rate of steering induced rate. Therefore, vehicle rollover can be avoided through steering system, active suspension technique, antiroll mechanism, and differential braking to reduce the yaw rate and roll angle generation rate [9].

Figure (4) shows a vehicle model for simulation of its road surface plane motion; the equations of motion using fixed axes system to the vehicle body which are given by the well-known half-car (bicycle model) [8]:

![Figure 4: Vehicle Model for Lateral Dynamics Simulation](image-url)
The vehicle deferential equations of motions are given for the lateral velocity $v_y$ and yaw angle $\Omega_z$ response at constant forward speed $v_x$ [10]. The longitudinal force balance can be written as follows:

$$m(\dot{V}_x - V_y \Omega_z) = F_{yf} \cos \delta_f + F_{xf} - F_{sf} \sin \delta_f \tag{11}$$

The lateral force analysis has the following form:

$$m(\dot{V}_y + V_z \Omega_z) = F_{yf} + F_{sf} \cos \delta_f + F_{xf} \sin \delta_f \tag{12}$$

The effective yaw torque can be written as a function in the front tyre slip angle and lateral effective force as:

$$I_z \dot{\Omega}_z = l_1 F_{sf} \cos \delta_f - l_2 F_{yf} + l_1 F_{xf} \sin \delta_f \tag{13}$$

In deriving the above equations of the model motions, it is assumed that the vehicle body is symmetric about its longitudinal plane [8]. Consequently, the vehicle lateral responses are functions in its geometry and mass distribution on their axles.

The slip angles of the axle tires can be defined in terms of vehicle motion variables yaw and lateral velocities, using small angle assumptions to get [11]:

$$\alpha_f = \delta_f - \frac{l_1 \Omega_z + V_y}{V_x}, \text{ and } \alpha_r = \frac{l_2 \Omega_z - V_y}{V_x}$$

The lateral forces acting on the front and rear tires are a function of the corresponding slip angle and cornering stiffness, and are expressed by

$$F_{yf} = 2C_{af} \alpha_f \text{ and } F_{yr} = 2C_{ar} \alpha_r$$

Assuming that the steer angle is small enough and the longitudinal force $F_{sf} = 0$, the equations of lateral and yaw motions of a vehicle with steer angle as the only input variable become [1], [12], [13]:

$$m \ddot{V}_y + m \frac{2l_1 C_{af} - 2l_2 C_{ar}}{V_x} \Omega_z + \frac{2C_{af} + 2C_{ar}}{V_x} V_y = 2C_{af} \delta_f(t) \tag{14}$$

$$I_z \ddot{\Omega}_z + \frac{2l_1 C_{af} - 2l_2 C_{ar}}{V_x} \Omega_z + \frac{2l_1 C_{af} - 2l_2 C_{ar}}{V_x} V_y = 2l_1 C_{af} \delta_f(t) \tag{15}$$

3.1. Vehicle body rolling motion

The roll of vehicle body involves a complex interaction of forces acting on the vehicle motion as influenced by the maneuver and road surface conditions. Vehicle roll is essentially caused by lateral inertial acting forces, which should be analyzed to avoid the vehicle rollover [11], [14]. The behavior of the vehicle body roll motion during its cornering negotiation is shown in Figure (5).
The roll moment of the vehicle body can be determined using one degree of freedom roll dynamics model as function in the vehicle body roll angle $\varphi$, suspension damping and stiffness parameters $C_\varphi$ and $K_\varphi$ as shown in Figure (4), where the roll motion moment equation can be determined as [15];

\[
I_x \ddot{\varphi} + C_\varphi \dot{\varphi} + (K_{pf} + K_{qr}) \varphi = mg_h \varphi + mh_y \dot{\varphi} \tag{16}
\]

Where $I_x$ is the vehicle body moment of inertia around its longitudinal axis, $\varphi$ is the vehicle body roll angle, $C_\varphi$ represent suspension damping coefficient on both sides, and $K_{pf}$ and $K_{qr}$, for both of the front and rear suspension [16].

**State Space Equations of the System**

The state space of the steering system and the vehicle lateral stability can be determined for the system as follows;

\[
\dot{x}(t) = Ax(t) + Bu(t) \tag{17}
\]

And the measurement equation for the system outputs $y(t)$,

\[
y(t) = Cx(t) + Du(t) \tag{18}
\]

Where the state variable vector $x(t)$ which includes the researched variables of the system in the time domain,

\[
x(t) = [\theta_s \ \theta_c \ x_r \ \delta_{wi} \ \delta_{wo} \ \nu_y \ \Omega_z \ \varphi \ \dot{\theta}_s \ \dot{\theta}_c \ \dot{x}_r \ \delta_{wi} \ \delta_{wo} \ \nu_y \ \Omega_z \ \dot{\varphi}]^T
\]
And the input matrix $B(t)$ of the primary excitation signal location and form due to the driver steering torque, it will be:

$$B(t) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ T_d/J_s \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

The input matrix $C(t)$ is used for identifying the measured variable $x(t)$ of the system response,

$$[C(t)] = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

And feedback of system nonlinearity is $D(t)$=0

The state matrix of the system $[A]$ can be determined as follows;
\[
\begin{align*}
A(t) &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 & a_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & a_{23} & 0 & 0 & 0 & 0 & a_{24} & a_{25} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 & a_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{41} & a_{42} & 0 & 0 & 0 & 0 & 0 & a_{43} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{51} & 0 & a_{52} & 0 & 0 & 0 & 0 & 0 & a_{53} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{61} & a_{62} & 0 & 0 & 0 & 0 & 0 & 0 & a_{63} & a_{64} & 0 & 0 & 0 \\
0 & 0 & 0 & a_{71} & a_{72} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{73} & a_{74} & 0 & 0 \\
0 & 0 & 0 & a_{81} & a_{82} & 0 & 0 & a_{83} & 0 & 0 & 0 & 0 & 0 & a_{84} & a_{85} & a_{86}
\end{bmatrix}
\]
\]

Where \(a_{11} = -ks/j_s\), \(a_{12} = ks/j_s\), and \(a_{13} = -cs/j_s\);
\(a_{21} = -k_1/j_e\), \(a_{22} = -ke/j_e\), \(a_{23} = kc/rp/j_e\), \(a_{24} = -c1/j_e\), and \(a_{25} = -ce/j_e\)
\(a_{31} = Ef*kc/rp/mr\), \(a_{32} = -Ef*kc/(rp*rp*mr) - 2*Eb*kst/(La*La*mr)\),
\(a_{33} = Eb*kst/La/mr\), and \(a_{34} = Eb*kst/La/mr\)

\(q = (rs + Rd*tan(sigm))*cos(sigm)*cos(taw)\), and \(Ms_k = Fz*i*cos(sigm)*sin(taw)\)*q;
\(eps_i = 1*pi/180\), \(fo = 0.02\), and \(ra = rs*cos(taw) + Rd*sin(sigm + eps_i)\);
\(M_r = Fz*i*fo*cos(taw)*ra\), and \(M_z = Fz*i*sin(sigm)*cos(taw)*q\)
\(M_z = Fz*i*sin(sigm)*cos(taw)*q\)
\(a_{41} = ksl/La/jw\), \(a_{42} = -(Mz_i + kw + ksl)/jw\), and \(a_{43} = -cw/jw\)
\(a_{51} = a_{41}\), \(a_{52} = -(Mz_o + kw + ksl)/jw\), and \(a_{53} = a_{43}\)
\(a_{61} = Cf/w/m\), \(a_{62} = a_{61}\), \(a_{63} = -2*(Cf/w+Crw)/m/Vx\),
and \(a_{64} = Vx - 2*(l1* Cf/w - l2* Crw)/m/Vx\);
\(a_{71} = l1* Cf/w/Iz\), \(a_{72} = a_{71}\), \(a_{73} = -2*(l1* Cf/w - l2* Crw)/Vx/Iz\)
and \(a_{74} = -2*(l1* Cf/w + l2* Crw)/Vx/Iz\);
\(SS = m_{sr}*hs/Ix\), \(a_{81} = SS* Cf/w/m\), \(a_{82} = a_{81}\), \(a_{83} = SS*g*(k-f+k'i)/l_x\), \(a_{84} = a_{63}*SS\); \(a_{85} = m_{sr}*hs/lx*((2*l2*Crw - 2*l1* Cf/w)/m/Vx/Vx), \(a_{86} = -Cph/Ix\)

4. Vehicle Steering and Stability Response

The response differences between the conventional and the widely used PD electric power steering dynamics due to its sensitivity to the high frequency response and damping it. Parameters variations will be studied to understand the vehicle lateral stability and body roll dynamics under effect of suitable excitation.

4.1 Steering system and stability data and parameters

To determine the steering system response with 8 DOF model shown in Figures (4,5), system data are given in Table (1)
Table (1): Data and parameters of electric power steering system [1], [12]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_d$</td>
<td>The steering torque of driver effort</td>
<td>1 Nm</td>
</tr>
<tr>
<td>$v_x$</td>
<td>Vehicle forward speed</td>
<td>25 km/h</td>
</tr>
<tr>
<td>$m_{hw}$</td>
<td>Masses of the steering hand wheel</td>
<td>1.5 kg</td>
</tr>
<tr>
<td>$R_{hw}$</td>
<td>Radius of the steering hand wheel</td>
<td>0.2 m</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Mass of the steering shaft</td>
<td>1 kg</td>
</tr>
<tr>
<td>$r_{sh}$</td>
<td>Radius of the steering shaft</td>
<td>0.02 m</td>
</tr>
<tr>
<td>$m_c$</td>
<td>Mass of the steering column</td>
<td>1.5 kg</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Radius of the steering column</td>
<td>0.03 m</td>
</tr>
<tr>
<td>$N$</td>
<td>The electric motor reduction ratio</td>
<td>20</td>
</tr>
<tr>
<td>$J_m$</td>
<td>Moments of the steering electric motor inertia</td>
<td>3.5/10000 kgm²</td>
</tr>
<tr>
<td>$R$</td>
<td>Motor armature winding resistance</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>$k_{e}$</td>
<td>The electric motor torque constant</td>
<td>0.05</td>
</tr>
<tr>
<td>$k_{b}$</td>
<td>Motor back electro-magnetic force constant</td>
<td>0.05</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Proportional factor of the controller</td>
<td>200</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Derivative factor of the controller</td>
<td>2</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Stiffness of the steering shaft</td>
<td>30 kNm/rad</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Stiffness factor of the steering column</td>
<td>33 kNm/rad</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Damping coefficient of the steering shaft</td>
<td>0.36 Nms/rad</td>
</tr>
<tr>
<td>$C_c$</td>
<td>Damping coefficient of the steering column</td>
<td>0.36 Nms/rad</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Damping coefficient of the electric motor</td>
<td>0.05 Nms/rad</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Radius of the steering pinion</td>
<td>0.02 m</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>Pinion to rack friction efficiency</td>
<td>98.5 %</td>
</tr>
<tr>
<td>$\eta_b$</td>
<td>Pinion to rack transmission efficiency</td>
<td>98.5 %</td>
</tr>
<tr>
<td>$K_{nl}$</td>
<td>The steering linkage stiffness</td>
<td>15000 Nm/rad</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Mass of the steering rack</td>
<td>5 kg</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Knuckle steering arm</td>
<td>0.1 m</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Rack axial damping coefficient</td>
<td>288.1 Ns/m</td>
</tr>
<tr>
<td>$m_w$</td>
<td>Vehicle road wheel mass</td>
<td>40 kg</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Radius of the vehicle road wheel</td>
<td>0.3 m</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Tire scrub radius</td>
<td>0.035 m</td>
</tr>
<tr>
<td>$r_a$</td>
<td>Wheel moment arm length</td>
<td>0.1 m</td>
</tr>
<tr>
<td>$M$</td>
<td>Vehicle mass</td>
<td>1500 kg</td>
</tr>
<tr>
<td>$L$</td>
<td>Vehicle wheelbase</td>
<td>2.5 m</td>
</tr>
<tr>
<td>$B$</td>
<td>Vehicle wheel track</td>
<td>1.34 m</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Vehicle roll arm</td>
<td>0.25 m</td>
</tr>
<tr>
<td>$m_{sr}$</td>
<td>Sprung mass of the tyre</td>
<td>1300 kg</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Vehicle front suspension stiffness</td>
<td>40 kN/m</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Vehicle rear suspension stiffness</td>
<td>40 kN/m</td>
</tr>
<tr>
<td>$C_{fw}$</td>
<td>Cornering stiffness of a front tire</td>
<td>75 kN/rad</td>
</tr>
<tr>
<td>$C_{rw}$</td>
<td>Cornering stiffness of a rear tire</td>
<td>80 kN/rad</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Tire lateral damping coefficient</td>
<td>88 Nms/rad</td>
</tr>
</tbody>
</table>
The inner front wheel turns at angle $\delta_1$, and the outer wheel turns at $\delta_2$ on corner turning radius $R$, at scrub radius $r_s$, and camber angle $\varepsilon$, angle of kingpin inclination $\sigma$. The wheel self-alignment torque depends on the tire lateral adhesion coefficient and its vertical force $F_z$, braking arm length $r_b$, the caster angle $\tau$, and the steering hand wheel with radius $R_{hw}$. The column has stiffness $K_c$ and damping coefficient $C_c$.

The following responses are given for a passenger car model to include its system steerability, and body roll motions for excitation of 1 Nm step torque of the driver hand steering at vehicle speed 90 km/h for uncontrolled steering system compared with Proportional Derivative PD controlled of electric assisted steering system.

### 4.2 Hand wheel response

Figure (6) shows the acceleration of the hand steer wheel for both steering systems in the time domain. Resolution of the obtained acceleration signals indicates that:

- Acceleration signals of the PD controlled steering system damps faster than that of the uncontrolled conventional steering system. It reached about $-38 \text{ rad/s}^2$ after about 20 ms, then it damps fast to around $-33 \text{ rad/s}^2$.

![Fig.(6): Acceleration of the driver hand wheel in the time domain](image)

- Uncontrolled steering system registered about $-50 \text{ rad/s}^2$ at high dynamic oscillations around 32 Hz as the natural frequency of the upper steering system. The signal damps slowly after about 250 ms, while the PD electric assisted system damps the system dynamics within 60 ms.
- An acceleration overshoot ratio of 15% registered the controlled system after 20 ms, while the uncontrolled system has overshoot of 51.5% after 15 ms.

### 4.3 STEERING RACK DYNAMICS
The steering rack dynamic motion without considerable clearances is given in the time domain for hand wheel unit step torque excitation in Figure (7).

![Fig.(7): Steering rack acceleration in the time domain](image1)

The given signals of the steering rack accelerations indicate that:

- The conventional steering system rack acceleration has a magnitude about $0.8 \text{ m/s}^2$ and vibrates about 9 frequent waves within about 250 ms.
- The PD controlled steering rack has an initial acceleration magnitude of about $0.07 \text{ m/s}^2$ then damps fast to its steady state.
- The rack acceleration of the conventional steering system is more about 11 times stronger more than that of the electric power assisted with PD control.

The steering rack inertial force is presented in Figure (8) in the time domain.

![Fig.(8): Steering rack force dynamics](image2)

Signal resolution of the given steering rack dynamics show that:
- The rack axial force of the conventional steering system vibrates between 290 N to -330 N which damps within 9 cycles in 275 ms.
- The PD controlled steering rack dynamic force vibrates between -20 N to +30 N through single initial wave due to high electric motor damping.

4.4 Road wheels dynamic response

The turning angle dynamics of the front wheel is presented in Figure (9), in the time domain at 90 km/h under steering and free rolling moments effects.

Analysis of the front steering wheel illustrates that:

- The road wheel lateral acceleration magnitude of the PD controlled steering system (0.75 rad/s²) is about 34% of that of the unassisted steering wheel (2.15 rad/s²) due to electric motor damping effect.
- The uncontrolled steering system is strong responsible to the tyre lateral stiffness vibration effect for around 30 Hz, which defects the tyre performance and high vehicle dynamics with low ride comfort.

![Fig. (9): Front wheel acceleration response for 1 Nm drive torque](image)

- These results ensure that the PD electric power steering assistance help to generate low steering system dynamics which ensures more tyre adhesion at long service life, good ride comfort and safety.

4.5 Vehicle body rolling rate

The vehicle body rolling rate over its suspension system is shown in Figure (10).
There is not more 5% differences of the vehicle body roll angle peaks between both of the steering systems, where the PD controlled steering generates the lower values after about 50 ms lag periods. The vehicle roll reached its quiet level after about 0.75 s according to its suspension parameters values.

The roll acceleration of the vehicle suspended body given in Figure (11) to understand the dynamic effects of the controlled electric assisted steering system on the vehicle roll motion and its reaction on the ride comfort and the vehicle motion safety.

Analysis of the given roll acceleration signals ensure that:

- The PD control of electric assisted steering generates a roll acceleration magnitude of about 70% of that unassisted steering system response.
- The uncontrolled system generates roll wave length of 0.25 s (4Hz) which is not comfortable where it lay in the human body critical frequency range.
- The PD controlled steering generates vehicle body roll acceleration wave length of 0.3 s (3.3Hz), which lies in the human comfortable range [4].

The vehicle torque of its roll motion is given in the Figure (12), which takes the roll acceleration waveform. The analysis of the roll torque waveform illustrates that:

- According to the vehicle suspension parameters and mass value, the conventional steering generates about 18 Nm roll torque of only 1 Nm on the hand steering wheel, which reduced to 12 Nm for the PD controlled steering.

Fig.(12): Roll torque of the vehicle suspended mass

- The vehicle body roll torque begins sharp as stringent sudden motion in the case of uncontrolled steering system, while it grows gradually in the case of the PD controlled steering which is safer.

4.6 Input torque effect on the vehicle body roll

There are many parameters can affect the vehicle lateral, yaw and body roll motions such as the steering input torque acting on the vehicle hand wheel, vehicle speed,….etc.

The PD controlled steering system generates the following vehicle roll torque in Figure (13)
The system response illustrates that:

- The vehicle body rolling registered torque signal magnitude of about 72 Nm (from -40 to +32 Nm) within 240 ms, which is related to 108 Nm of the uncontrolled steering for the same natural frequencies according to the suspension parameter values.
- The system response damps fast at natural frequency nearby 2.7 Hz, which lays out of the human uncomfortable frequency range.
- All peaks of the responses are in proportion to the driver input torque.

5. Conclusions

From the obtained analytical results of the vehicle steering dynamics, it can be conclude that:

- The power assisted steering system on the steering column with PD control is more comfortable to the driver with very low signal overshoot at relative low natural frequencies and safe for the steering system for low performance time compared to the unassisted steering system.
- Use of the electric assisted steering system with PD control helps to avoid stringent rack dynamics at 1/11 acceleration ratio of the unassisted steering, which keeps the rack in low dynamic performance for longer service life.
- The road wheel lateral acceleration magnitude of the PD controlled steering system is about 34% of that unassisted steering wheel due to electric motor damping effect, where the unassisted road wheel vibrates at 30 Hz strongly.
- The uncontrolled system generates roll wave length of 250 ms (4Hz) which is not comfortable where it lays within the human body critical frequency range, while the PD controlled steering system generates vehicle body roll acceleration wave length of 300 ms (3.3Hz), which lies within the human comfortable frequency range.
- These results ensure that the PD electric power steering assistance help to generate low steering system dynamics which ensures more tyre adhesion at long service life, good ride comfort and safety for vehicle long service life.
The electric assisted steering system registered lower frequency response, lower rolling moment and tyre steering acceleration. All of them give vehicle more service safety which increases the vehicle quiet service life.

The steering joint friction are relatively small compared to that of the pinion with rack, which given in Eq. 6.

The roll motion is a direct response of the vehicle lateral acceleration in Eq.16, and these relations are given in the student search.

The first roll wave duration is about 0.25 ms (4 Hz) for the uncontrolled steering, Fig. 11, which took about 0.3 ms (3.3 Hz), where the human body response is very sensitive within 4-8 Hz. Consequently, the vehicle vibration generated signals should be avoided in the given range [4].

This paper is projected to analyze the improvement in the vehicle body roll motion when we use electric assisted steering system and its effect on the vehicle safety.