ENGINEERING RESEARCH JOURNAL (ERJ)
Vol. 1, No. 50 October 2021, pp.7-13
Journal Homepage: erj.bu.edu.eg

Uncertainty Evaluation in a Single Sample Experiment of Centrifugal Pump

Mazen A. Alfuraih and Khaled H. Aldoseri

Public Authority for Applied Education and Training

Subah Al Salem Industrial Institute-Kuwait

Abstract: The present paper presents a systematic evaluation process of experimental work of uncertainty analysis. The experiment is considered a system composed entirely as a function of system method to obtain certain data. A systematic approach is set to estimate the uncertainty in these data in an arranged steps for rotary pumps such as centrifugal radial flow, axial flow, and mixed flow pumps. The paper presents the important values which should be carefully measured to obtain an accurate uncertainty value for laboratory tests and also, field tests for pump systems arrangement.

1. Introduction

An important question in all experimental engineering work is the accuracy of the results. Even an otherwise good report loses much of its significance. It is important to estimate the uncertainty for different reasons. Since the readers will usually be unfamiliar with both the equipment and the techniques employed.

The purpose of this paper is to provide a simple and systematic method of estimating experimental uncertainty. Note that the uncertainty and not the error is estimated. The uncertainty is a possible value of the error. It thus describes our lack of knowledge about the true value of a measured quantity. In most biological, medical, and chemical work experiments are repeated and controls are employed so that a statistical measure of the error can be made. In most engineering experiments this is not possible due to time and cost requirements and to the complexity of the experiments. In a large part, this difficulty can be overcome by estimating the experimental uncertainty. The concept of uncertainty is therefore fundamental to all that follows. The error in a given reading is the true value minus the observed value and is just a fixed number. The error in a single observation cannot be treated statistically. But the uncertainty, or what we think the error might have been in this same observation, is a variable; it can take on any value from zero to a large number. The uncertainty can be treated in part by the methods of statistics. This basis for the treatment of the accuracy of engineering experiments is discussed in Refs.[1] to[5].

Any procedure for calculating the uncertainty in the results consists of three steps. These steps are:

1. Estimate and record in concise form the uncertainty in each of the variables. Variable here means a quantity observed (recorded) directly in the laboratory.

2. Calculate the uncertainty in the result due to the uncertainty in each of the variables.

3. Combine the uncertainties found in step 2 to give the total uncertainty in the result. (If there are intermediate results, then step 2 and 3 must be repeated to obtain the final result.)

2. PRELIMINARY DISCUSSION

First, we will discuss the nature of uncertainties in the variables. Uncertainties arise from possibilities for error. Errors can be classified as: fixed errors, random errors, and human errors.

Fixed errors are errors which are constant for a given procedure. Fixed errors often arise from the inherent constructionof the observing instrument. For example, a balance might have one arm too long so that it always gives readings which are slightly too high. Such an error could not be detected, no matter how many times observations were repeated on this balance. Fixed errors in the result may also arise from the use of an approximate theory in calculating the result from the data.

Random errors are those errors which vary from reading to reading. An example might be random noise in an electronic Instrument. A useful rule which can be derived from statistics is that accidental errors can be reduced approximately as the square root of the number of readings by repeating and averaging the results.

Human errors are those attributable to mistakes by the observer rather than sampling or instrumentation. Examples are misreading a scale or reading the wrong dial.

The possibility or error in each of these classes contributes to the uncertainty in a given observation, and all of them must therefore be considered. The best thing to do would be to repeat all readings using enough different sampling techniques, instruments, and observers so that the entire uncertainty in a given observation could be calculated by statistical means. This should be done where possible, but from the practical point of view, the engineer almost never has the money or the time. Fortunately, one can usually estimate the uncertainty well enough to serve his purposes. These estimates will be correct to perhaps $\pm$ 50 per cent of the uncertainty. They can be based on experience, judgment, and auxiliary experiments as well as the operation of the instruments during the experiment. Uncertainties due to sampling as well as those due to transferring the impress of the sample to some scale device and the reading of the scale must be allowed for. Note that both the sampling process and the transfer process may give rise to either fixed or random errors or both.In order to illustrate the sources of error let us examine the measurement of pressure with a static tap and manometer. Errors may be due to improper tap construction, leaks in the pressure tubing or joints, imperfect construction of the manometer, or reading of the manometer scale.A useful rule of thumb for the instrument uncertainty alone is that the instrument uncertainty should be of the order of one-half the smallest scale division of the instrument, but not all instruments obey this rule.

3. DESCRIPTION OF UNCERTAINTY IN MEASURED QUANTITIES

Some method is needed for recording the estimates of uncertainties in a meaningful concise form. Many authors have used the so-called "maximum uncertainty" which is a value that the error will never exceed. But all available experiments show that when many readings are taken a few will always have very large errors, e.g., it is always possible that the instrument is broken completely. It is, therefore, better to use an uncertainty interval based on suitable odds, See Ref [1] toRef [3] for a complete discussion. This is done as follows:

Pressure = 103 + 1.3kPa (20 to 1) (1)

In Equation (1) the first number,103kPa is the mean of the readings and represents the best estimate of the observed value. The second number has been defined in Ref. [4]and Ref. [5]as the "uncertainty interval. The numbers in parentheses indicate that the experimenter would be willing to bet 20 to l that the true value lies within plus or minus the uncertainty interval of the best estimate. In this case,one would be willing to bet 20 to 1 that the true value is between 101.7 and 104.3kPa. A more precise way of saying this would be: "I believe that if this reading were repeated twenty-one. times, by more than one observer and using more than one kind of instrument, twenty of the readings would be between 101.7 and 104.3 kPa." in experimental work the engineer usually wants his odds to be at least 10 or 20 to l. The engineer can set his odds suitably for each experiment but must take them to be the same throughout at least one complete calculation in order to get answers that are useful.

4. EFFECT ON RESULT OF UNCERTAINTY IN ONE VARIABLE

The effect of the uncertainty in each variable on the uncertainty of the result must now be found. If we denote the result by R, a variable by Vi the uncertainty interval of the variable as wi, and the uncertainty interval in the result due to wi alone as w_{Ri}, then:

$$\delta R \cong \frac{\partial R}{\partial V_i}\delta V_i \qquad (2)$$

And

$$w_{R_i} = \frac{\partial R}{\partial V_i} w_i \qquad (3)$$

Dividing both sides by R (and recognizing δR/R as ∂(lnR)):

$$\frac{w_{R_i}}{R} = \frac{\partial R}{\partial V_i}\frac{w_i}{R} = \frac{\partial R}{\partial V_i}\frac{w_i}{R}\times\frac{V_i}{V_i} = \frac{\partial(\ln R)}{(\ln V_i)}.\frac{w_i}{V_i} \qquad (4)$$

Note that Eq. (3) shows that the uncertainty in R due to a unit uncertainty in the variable Vi is ∂R/∂Vi, while Eq. (4) shows that the per cent uncertainty in R due to a 1 per cent uncertainty in Vi is ∂(lnR)/∂(ln Vi). The ln form is almost always more useful and simpler in actual calculations.

5. COMBINED EFFECT OF MANY VARIABLES ON UNCERTAINTY OF THE RESULT.

If there is more than one variable affecting the result, then Eq. (2) becomes:

$$\delta R = \frac{\partial R}{\partial V_1}\delta V_1 + \frac{\partial R}{\partial V_2}\delta V_2 + \ldots\ldots + \frac{\partial R}{\partial V_n}\delta V_n \qquad (5)$$

Students who have had statistics will recognize that the uncertainty interval is simply a guesstimated confidence interval.

A similar addition of the uncertainty intervals is not valid, however, since each w can be either positive or negative and it is very unlikely that all of them will be positive for anyone given reading. This problem or signs can be taken into account or by the use of statistics. It is shown in Ref. [3] that a valid approximation is to add the quantities given by Eq. (3) or Eq. (4) by the Square so long as the uncertainties in each of the variables are independent. This can be expressed in terms of formulae, for n independent variables, as:

$$w_R = \sqrt{\left(\frac{\partial R}{\partial V_1}w_1\right)^2 + \left(\frac{\partial R}{\partial V_2}w_2\right)^2 + \ldots\ldots + \left(\frac{\partial R}{\partial V_n}w_n\right)^2} \qquad (6)$$

$$\frac{w_R}{R} = \left\{\left(\frac{\partial(\ln R)}{\partial(\ln V_1)}\frac{w_1}{V_1}\right)^2 + \left(\frac{\partial(\ln R)}{\partial(\ln V_2)}\frac{w_2}{V_2}\right)^2 + \ldots\ldots + \left(\frac{\partial(\ln R)}{\partial(\ln V_n)}\frac{w_n}{V_n}\right)^2\right\}^{1/2} \qquad (7)$$

Two methods are available for evaluating the ∂R/∂V or ∂(ln R)/∂(ln V) terms. The first is analytically. Suppose, for example, that the result is the kinetic energy per unit mass then,

$$K.E. = \tfrac{1}{2}\rho V^2 \qquad Then \begin{cases} R = K.E. \\ \rho = V_1 \\ V = V_2 \end{cases} \qquad (8)$$

Differentiating

$$d(KE) = dR = \underbrace{\tfrac{1}{2}V^2}_{\frac{\partial R}{\partial \rho}}\,\mathrm{d}\rho + \underbrace{\rho \mathrm{V}}_{\frac{\partial R}{\partial V}}\mathrm{dV} \qquad (9)$$

on dividing by Eq. (8)

$$dR = 1\ \frac{\mathrm{d}\rho}{\rho} + 2\ \frac{\mathrm{dV}}{\mathrm{V}} \qquad (10)$$

$$\frac{\partial(\ln R)}{\partial(\ln \rho)} = 1\ , \qquad \frac{\partial(\ln\ R)}{\partial(\ln\ V)} = 2$$

Hence

Note that Eq. (10), the nondimensional form, could have been found by direct logarithmic differentiation on Eq. (8). The nondimensional form of Eq. (10) is not only simpler in many cases, but it gives directly what the experimenter usually wants, the percent uncertainty interval in the result. The result of Eq. (9) can be found by a second method: $\frac{\partial(\ln R)}{\partial(\ln \rho)} = 1$, $\frac{\partial(\ln\ R)}{\partial(\ln\ V)} = 2$

Substitute $V + \delta V$ for V and $\rho + \delta\rho$ for ρ into Eq. (8) gives:

$$R + \delta R = \tfrac{1}{2}(\rho + \delta\rho)(V + \delta V)^2$$
$$= \tfrac{1}{2}\rho V^2 + \tfrac{1}{2}V^2\delta\rho + \rho V\delta V + V(\delta\rho)V + \tfrac{1}{2}\rho(\delta V)^2 + \tfrac{1}{2}\delta\rho(\delta V)$$

Then subtract the original eq. (8) and neglect terms containing $(\delta)^2$ or higher powers of δ. Therefore, again finds:

$$\delta R = \tfrac{1}{2}V^2\delta\rho + \rho V\delta V \qquad (9)$$

Equation (10) can again be obtained by dividing Eq. (9) by Eq. (8). This second technique not only allows the nature of the approximation made but also can be employed where only a graphical relation between R and the variables is known.

The results of Eq. (9) or Eq. (10) can now be expressed as uncertainty intervals for the result by putting Eq. (9) in the Eq. (6) by substituting the uncertainty intervals for the δ terms.

This gives:

$$w_R = \sqrt{\left(\tfrac{1}{2}V^2 w_\rho\right)^2 + \left(\rho V w_V\right)^2} \qquad (11)$$

and by Eq. (7), Eq. (10) similarly becomes:

$$\frac{w_R}{R} = \sqrt{\left(\frac{w_\rho}{\rho}\right)^2 + \left(\frac{w_V}{V}\right)^2} \qquad (12)$$

6. SAMPLE FORMULAE

For the student who is not familiar with log differentiation it may be helpful to convert the equation to an equivalent log form before differentiating. For example,

If $\quad R = f(x, y, z) = x^2\ y\ z^3$

then $\quad \ln R = 2 \ln x + \ln y + 3 \ln z$

In this way In R is expressed as an explicit function and its partial derivative with respect to the ln of any one of the variables can easily be evaluated.

The student should prove to his own satisfaction that the following common error relations are correct .For

$$R = V^n, \qquad \frac{\delta(\ln R)}{\delta(\ln V)} = n$$
$$R = e^V, \qquad \frac{\delta(\ln R)}{\delta(\ln V)} = v \qquad (13)$$
$$R = \ln V, \qquad \frac{\delta(\ln R)}{\delta(\ln V)} = \frac{1}{\ln V}$$
$$R = \sin V, \qquad \frac{\delta(\ln R)}{\delta(\ln V)} = V \cot V$$

and

$$R = x \pm y, \qquad \frac{\delta(\ln R)}{\delta(\ln x)} = \frac{x}{x \pm y}$$
$$R = xy^n \pm z^m, \frac{\delta(\ln R)}{\delta(\ln x)} = \frac{xy^n}{R}$$
$$R = xy^n \pm z^m, \frac{\delta(\ln R)}{\delta(\ln y)} = \frac{nxy^n}{R}$$
$$R = xy^n \pm z^m, \frac{\delta(\ln R)}{\delta(\ln z)} = \frac{mz^m}{R}$$

An examination of these equations will show that sometimes the percentage uncertainty in the result (w_{R_i}/R) is greater than the percentage uncertainty in the basic quantity Vi and sometimes less. Note particularly the effect of whethern >l or n < 1 in Eq. (13), R = V^n. This is a very common case occurring in such formulas as area = (length)^2, K.E. = 1/2(mV^2), etc. Equation (13) demonstrates a useful property of logarithmic plotting; namely, the same scalar distance on a logarithmic plot represents the same percentage uncertainty anywhere on the graph. Equation (13) shows why data based on the difference between two numbers of approximately equal size is inherently inaccurate, Equation (14) is simply an application of the basic ideas of Eqs. (13) and (14) given so that the student can check his ability to work out such cases.

The researcher should work out for himself several examples. Before doing this, it may be helpful to study the example which is given at the end of this paper

7. SUMMARY OF METHOD SUGGESTED

A- Summarizing the method suggested here, we then have:

1. Estimate and record the uncertainties as:

Mean + (uncertainty interval), (odds of m to l), and convert to percent uncertainty interval

2.Compute the uncertainty in the result from the equation relating the measured quantities to the result and using the relations:

$$\frac{w_R}{R} = \left\{ \left(\frac{\partial(\ln R)}{\partial(\ln V_1)} \frac{w_1}{V_1} \right)^2 + \left(\frac{\partial(\ln R)}{\partial(\ln V_2)} \frac{w_2}{V_2} \right)^2 + \dots\dots + \left(\frac{\partial(\ln R)}{\partial(\ln V_n)} \frac{w_n}{V_n} \right)^2 \right\}^{1/2} \qquad (15)$$

Or

$$w_R = \left[\left(\frac{\partial R}{\partial V_1} \frac{w_1}{V_1} \right)^2 + \left(\frac{\partial R}{\partial V_2} \frac{w_2}{V_2} \right)^2 + \dots\dots + \left(\frac{\partial R}{\partial V_n} \frac{w_n}{V_n} \right)^2 \right]^{1/2} \qquad (16)$$

3.If step 2 gives intermediate results, repeat to obtain the final result.

B- Consideration or the uncertainty in the result, using some method such as that suggested here, is essential to the planning and execution of useful engineering experiments. (14)

8- IMPORTANT COMMENTS

Propagation effects

It is perhaps most important or all to note that a one percent error in reading will not necessarily give a one-percent error in the result. A one-percent uncertainty may give such a one-percent error in the result, but it may also give a 100-percent error or a 1/100-percent error. It all depends on the nature of the function connecting the measured quantities and the result.

It is also very important to realize that when quantities are added by the square, as occurs in the propagation formulae, only the large terms contribute significantly to the result. Keeping in mind that each term in the equation is of the form $\partial(\ln R)/\partial(\ln V_i)(w_i/V_i)$, and not just (w_i/V_i), it is clear that the accuracy of an experiment can be improved significantly only by reducing the large terms. Whittling away at the small ones will produce only wasted effort. For example, consider: 5 + 1 + 1 + 1 + 1 + 1 = 10. But [25+12+12+12+12+12]$^{1/2}$ = (30)$^{1/2}$ = 5.48

Consequently, reducing all five of the unity terms to zero error in this case would yield on the average only a ten-percent improvement in the total accuracy of the result. A good rule of thumb follows: "If anyone term in the propagation formula is less than one-fifth of the largest term, it can be neglected." (By term we still mean $\partial(\ln R)/\partial(\ln V_i)(w_i/V_i)$.

9. Use of the Method

Obviously, the technique described above applies to an experiment that has been performed; that is what we have set it down for.

But from the comments just made on propagation. It can be seen that; it is quite essential to use this technique in considering how a given experiment can be made moreaccurate. And the most important time for such a considerationis in the design stage. Very frequently the difference between success and failurein an entire experiment is the proper understanding and application of the ideas given above in thedesign stage. Clearly, it is easier to change the design on paper than after has been built. Application of these ideas in advance, based on a proposed experimental method, will not only clarify the proposed experiment, and improve itsaccuracy, but it will also goa long way toward eliminating the "hopeless experiment" that results,all too frequently, from the construction of the constructing of the first thing that comes to mind. Such an analysis will also pinpoint in advancethose measurements which are critical, and which may need specialattention. Furthermore, even if it is intended to replicate, that is to repeat readings, in the real experiment,it's clearthat no replication is possiblein the design stage. Hence the technique described above is the only one that can be used in the design stage the only one single apparatus. Therefore: let us repeat: THE MOST IMPORTANT TIME TO ANALYSE THE UNCERTAINTIES IS IN THE DESIGN STAGE. Experience for such use can 'be gained by uncertainty analyses on laboratory experiments. Systematic Techniques for use in the design are given in Ref[4] and Ref. [5]. Useful references are listed from [7] to [14].

10. DESIGN of INSTRUMENTS

A consideration of the output accuracy of an instrument, including all the possible sources for error inside it, will show that the problem is mathematically entirely analogous to the one we have just discussed. It has been shown in recent work, unfortunately not published, that the application of these ideas is not only helpful but also financially profitable. To state this more clearly, if it is desired to build an instrument of a given output, accuracy, the cost can be minimized by the use of the technique described above plus knowledge of the cost of reducing the uncertainty in each element of the instrument or machine. In complex, instruments with many circuits (and or parts) such an analysis have created cost savings of very large magnitudes.

11. PRACTICAL EXAMPLE RESULTS

Estimating the uncertainty in the centrifugal Pump experiment: For the Pump the measured final quantities are: -

R.p.m.; N;

Total Head; $H = \Delta\left(\dfrac{P}{\rho g} + \dfrac{V^2}{2g} + Z\right)$;

Flow rate;

$$Q = m^3/\sec = C_D A\sqrt{\Delta P/Q} = C_D A\sqrt{\Delta P/\rho}$$

Brake Power; $BP = \dfrac{2\pi rFN}{\text{constant}}$

and Pump total efficiency; $\eta_p = \dfrac{\rho g Q H}{BP}$.

In the following table the basic quantities involved with reasonable magnitudes of uncertainty intervals for 20 to 1 odd in each of the quantities actual measured in the experimental yields:

Measured Quantity (20 to 1)	Magnitude + w Interval. (20 to 1)	Percent Uncertainty
RPM	3,760 + 4 rpm	+0.11
Dynamometer Load	155 + 0.9 N	+0.57
Torque Arm	320 + 0.25 mm	+0.08
$\Delta P_{orifice}$	150 + 1.25 mmHg	+0.83
ΔZcorr.	2.7 + 0.045m	+1.67
Orifice Area and CD	----------------	+ 0.8*

The uncertainty intervals for the results can now be calculated:

Shaft Brake Power:

The formula for shaft horsepower is:

$$BP = \frac{2\pi rFN}{\text{constant}}$$

Hence

$$\frac{w_{SBP}}{BP} = \sqrt{\left(\frac{w_r}{r}\right)^2 + \left(\frac{w_n}{n}\right)^2 + \left(\frac{w_F}{F}\right)^2} = \sqrt{(0.08)^2 + (0.11)^2 + (0.57)^2}$$

$$\frac{w_{BP}}{BP} = 0.586$$

Note here that percentage uncertainties combine directly only because the equation is linear in form.

Flow Rate:

The formula for flow is: $Q = C_D\sqrt{\Delta p/\rho}$; Hence

$$\frac{w_Q}{Q}=\sqrt{\left(\frac{w_{C_D}A}{C_DA}\right)^2+\left(\frac{1}{2}\frac{w_{\Delta P}}{\Delta P}\right)^2}=\sqrt{(0.8)^2+(0.4)^2}$$

$$\frac{w_Q}{Q}=0.894$$

From an examination of the flow vs ΔP orifice graph at the test site and the tolerances specified in the Egyptian Ministry of Industry for Fluid Meters Report, 1970.

Note here that $w\rho/\rho$ is negligible. Also, note that $w_{\Delta p/p}$ has the coefficient of $\partial \ln Q/\partial \ln \Delta P = 1/2$ because the equation was not linear.

Total Head By definition, $H=\Delta\left(\frac{P}{\rho g}+\frac{V^2}{2g}+Z\right)$, Hence

$$dH=d\left(P/\rho\right)+d\left(V^2/2g\right)+dZ$$

And

$$\frac{w_H}{H}=\sqrt{\frac{\left(w_{P/\rho}\right)^2+\left(w_{V^2/2g}\right)^2+\left(w_Z\right)^2}{H^2}}$$

Computing each term separately as an intermediate result

$$w_Z=0.045\ m$$

$$\frac{V^2}{2g}=\frac{1}{2g}\left(\frac{Q}{A}\right)^2$$

$$\frac{w_{V^2/2g}}{V^2/2g}=\sqrt{\left(2\frac{w_Q}{Q}\right)^2+\left(2\frac{w_A}{A}\right)^2}$$

Since w_A/A is less than one-fifth of w_Q/Q, $\left(w_A/A\right)^2$ can be neglected. Then:

$$w_{V^2/2g}=\frac{V^2}{2g}\frac{w_Q}{Q}$$

$$\frac{w_{V^2/2g}}{2g}=0.000918$$

$w_P=w_{P/\rho}=0.036$ (Since w_ρ is negligible)

Thus, combining the intermediate results, we obtain the percent uncertainty in H as

$$\frac{w_H}{H}=\sqrt{\frac{(0.036)^2+(0.000918)^2+(0.045)^2}{H^2}}=\frac{0.057}{H}$$

for H = 30m; w_H/H = 0.19 per cent. But for H = 9m; w_H/H = 0.63 per cent.

Note also that if the calibrated Bourdon gage has been used instead of the pressure weigher w_H=0.54 and for H= 9m,the uncertainty would have been w_H/H = 6. per cent. This fact is what dictated the use of the more cumbersome pressure weigher in this set-up.

12. Pump Efficiency

By definition pump efficiency, η, is: $\eta_p=\frac{\rho QH}{BP}$

Since the equation is linear we have:

$$\frac{w_{\eta_p}}{\eta_p}=\sqrt{\left(\frac{w_Q}{Q}\right)^2+\left(\frac{w_\rho}{\rho}\right)^2+\left(\frac{w_{BP}}{BP}\right)^2+\left(\frac{w_H}{H}\right)^2}=\sqrt{(0.89)^2+(0)^2+(0.58)^2+(0.63)^2}$$

$$\frac{w_{\eta_p}}{\eta_p}=1.24$$

Tabulating

Result	Percent UncertaintyInterval 20 to 1
BP	0.6
Flow Rate	0.9
Total Head	0.19 - 0.63
Pump Efficiency	1.24

These results would usually be sufficient information to present in a report. It shows that the centrifugal Pump is extremely well instrumented.

13. REFERENCES

[1] Airy, Sir George Biddell: "Theory of Errors of Observation,"Macmillan and co: London, 1879.

[2] Deiderichs and Andrae; "Experimental Engineering," JohnWiley and Sons, New York, 1930, Chạp. 1, Vol. 1.

[3] Kline, s. J. and McClintock, F. A.; The Description ofUncertainties in Single Sample Experiments, MechanicalEngineering, January 1953. to Wilson, W. A. "Design of Power-Plant Tests to InsureReliability of Results," Trans, ASME, Vol. 17, No. 4, May 1955.

[4] M.G.Higazy; “Analysis and Accuracy of Experimental Methods”,Int JPetrochemRes. 2019; 3(1): 262-268.Doi: 10.18689/ijpr-1000145

[5] M. G. Higazy. Experimental Measurement Theory. Cairo, CR: Dar- Elmaarifa Publisher; Jan. 2019.

[6] JCGM 200: International Vocabulary ofMetrology (VIM) – Basic and general conceptsand associated terms, 2008.

[7] JCGM 100: Evaluation of measurement data– Guide to the expression of Uncertainty inMeasurement, 2008.

[8] JCGM 101: Evaluation of measurement data– Supplement 1 to the “Guide to theexpression of Uncertainty in Measurement” –Propagation of distributions using a MonteCarlo method, 2008.

[9] .JCGM 102: Evaluation of measurement data– Supplement 2 to the “Guide to theexpression of Uncertainty in Measurement” –Extension to any number of output quantities,2011.

[10] .Sydenham P H, Handbook of MeasurementScience. Volume 2. Practical Fundamentals(John Wiley & Sons), 1983.

[11] .Ower E, Pankhurst R C, The Measurement ofAir Flow (Oxford, Pergamon), 1977.

[12] .Sabersky R H, Acosta, A J, Hauptmann, E G,Gates, E M, Fluid Flow (New Jersey, PrenticeHall), 1999.

[13] .NIST, CODATA Internationally recommended2018 values of the Fundamental PhysicalConstants,https://physics.nist.gov/cuu/Constants